Welcome to AS Calculus AB. I hope that you are having a great summer and are starting to think about next year. As you know, Calculus AB is a college level mathematics class, and a certain level of prerequisite knowledge is required. The problems below are meant to be a warmup for the year, with topics pulled primarily from traditional Algebra 2 and Precalculus classes, and they represent some of the ideas and skills that should be fluent with as we begin the course in September. I would recommend doing this sometime in the weeks before school starts so that you are ready to go on day 1 of class. Feel free to email me if you have questions about the problems.

1. In 2007, the world’s population reached 6.7 billion people. In 2008, it was 6.78 billion.
   a. Let’s assume the population is growing linearly. Write a linear function that could give the population as a function of time, \( t \), in years after 2007.
   b. Now let’s assume the population is growing exponentially. Note that 6.078 billion is 1.2% more than 6.7 billion. Write an exponential function that could give the population as a function of time, \( t \), in years after 2007.
   c. Without a graphing calculator or Desmos, sketch a rough graph of these two models.
   d. Use both to estimate the population of the world in the year 2026.
   e. When does each predict that the world will reach 10 billion people?
   f. Which of the two models do you think is more reasonable? Why?

2. Find formulas for the functions defined below.
   a. A parabola opening downward with a vertex at \((2,5)\). Extra challenge: make it pass through the point \((1,1)\).
   b. The bottom half of a circle centered at \((4,0)\) with a radius of 4.
   c. A cubic polynomial having \(x\)-intercepts at \(x = 1, 5, 7\) and a \(y\)-intercept at \(y = 2\).
   d. A cosine function with a maximum at \((0,5)\), a minimum at \((\pi, -3)\) and no maxima or minima in between.
   e. A rational function with a vertical asymptote at \(x = 2\), a horizontal asymptote at \(y = 1\), an intercept at \(x = 0\), and a hole at \(x = 4\).

3. These equations are all solved for \(B\) in terms of \(A\). Solve them for \(A\) in terms of \(B\) instead. Example: \(B = A^2 + 3 \rightarrow B - 3 = A^2 \rightarrow A = \pm \sqrt{B - 3}\).
   a. \(B = 3 \log_4 (A + 2)\)
   b. \(B = A^2 + 6A\)
   c. \(B = \frac{1}{(\sin A + 1)^2}\)
   d. \(B = 2^A + 2^A\)
   e. \(B = |A + 1|\)
4. On the same axes, sketch graphs of the following functions. You can use Desmos or a calculator to check your work, but draw a sketch by hand first.

On one set of axes:  
\[
y_1 = \sqrt{x} \\
y_2 = \sqrt{x - 4} \\
y_3 = \sqrt{x - 4} \\
y_4 = -4\sqrt{x}
\]

On another set of axes:  
\[
y_1 = \ln x \\
y_2 = \ln(x - 5) \\
y_3 = \ln x - 5 \\
y_4 = -5\ln x
\]

On another set of axes:  
\[
y_1 = \cos x \\
y_2 = \cos(x - \pi) \\
y_3 = \cos x - \pi \\
y_4 = -\pi \cos x
\]

5. Graph the following function by hand.

\[
f(x) = \begin{cases} 
  x^2 - 10, & \text{if } x \leq 2 \\
  x + 2, & \text{if } 2 < x \leq 4 \\
  4, & \text{if } 4 < x
\end{cases}
\]

6. Let \( f(x) = \frac{1}{x+2} \) and \( g(x) = x^2 \).
   
a. Show that \( f(g(7)) = \frac{1}{51} \).  
b. Show that \( g(f(7)) = \frac{1}{81} \).
   
c. Find \( f(g(x)) \).  
d. Find \( g(f(x)) \).
   
e. Find \( g\left(g\left(g\left(g\left(g(x)\right)\right)\right)\right) \).  
f. Find \( f(p + 2) \) and \( g(p + 2) \).

7. For the function \( y = x^2 \)...
   
a. Find the slope of the line segment between the points on the function at \( x = 2 \) and \( x = 5 \).
   
b. Find the length of that segment.
   
c. Find the equation of the line that bisects this segment perpendicularly.
   
d. Now find the slope of the line segment between \( x = 2 \) and \( x = 2.1 \).
   
e. Now find the slope of the line segment between \( x = 2 \) and \( x = 2.01 \).
   
f. Now estimate the slope of the tangent line at \( x = 2 \). Draw a picture of the lines from a, d and e if you are wondering how you are supposed to know this answer.